

Properties of Equality - Equations are solved using the Properties of Equality.

Addition and Subtraction Properties of Equality	For any real numbers a , b , and c , if $a = b$, then $a + c = b + c$ and $a - c = b - c$.
Multiplication and Division Properties of Equality	For any real numbers a , b , and c , if $a = b$, then $a \cdot c = b \cdot c$ and, if $c \neq 0$, $\frac{a}{c} = \frac{b}{c}$.

Example 1: Solve $10 - 8x = 50$.

$$\begin{aligned} 10 - 8x &= 50 && \text{Original equation} \\ 10 - 8x - 10 &= 50 - 10 && \text{Subtract 10 from both sides.} \\ -8x &= 40 && \text{Simplify.} \\ x &= -5 && \text{Divide both sides by } -1. \end{aligned}$$

Example 2: Solve $4x + 5y = 100$ for y .

$$\begin{aligned} 4x + 5y &= 100 && \text{Original equation} \\ 4x + 5y - 4x &= 100 - 4x && \text{Subtract } 4x \text{ from both sides.} \\ 5y &= 100 - 4x && \text{Simplify.} \\ y &= \frac{1}{5}(100 - 4x) && \text{Divide both sides by 5.} \end{aligned}$$

Solve each equation and check your solution.

1. $3s = 45$

$$\boxed{s = 15}$$

2. $17 = 9 - a$

$$\begin{aligned} -a &= 8 \\ \boxed{a} &= \boxed{-8} \end{aligned}$$

3. $5t - 1 = 6t - 5$

$$\begin{aligned} 4 &= t \\ \boxed{t} &= \boxed{4} \end{aligned}$$

4. $\frac{2}{3}m = \frac{1}{2}$

$$\begin{aligned} \left(\frac{3}{2}\right) \frac{2}{3}m &= \left(\frac{3}{2}\right) \cdot \frac{1}{2} \\ \boxed{m} &= \boxed{\frac{3}{4}} \end{aligned}$$

5. $-8 = -2(z + 7)$

$$\begin{aligned} -8 &= -2z - 14 \\ 6 &= -2z \\ \boxed{z} &= \boxed{-3} \end{aligned}$$

6. $3x + 17 = 5x - 13$

$$\begin{aligned} 30 &= 2x \\ \boxed{x} &= \boxed{15} \end{aligned}$$

7. $120 - \frac{3}{4}y = 60$

$$\begin{aligned} -\frac{4}{3} \left(120 - \frac{3}{4}y\right) &= -\frac{4}{3} \cdot 60 \\ \boxed{y} &= \boxed{80} \end{aligned}$$

8. $\frac{5}{2}n = 98 - n$

$$\begin{aligned} \frac{5}{2}n + \frac{2}{2}n &= 98 \\ \frac{7}{2}n &= 98 \\ \boxed{n} &= \boxed{28} \end{aligned}$$

9. $4.5 + 2p = 8.7$

$$\begin{aligned} 2p &= 4.2 \\ \boxed{p} &= \boxed{2.1} \end{aligned}$$

Solve each equation or formula for the specified variable.

10. $a = 3b - c$, for b

$$\begin{aligned} 3b &= a + c \\ \boxed{b} &= \boxed{\frac{a+c}{3}} \end{aligned}$$

12. $2xy = x + 7$, for x

$$\begin{aligned} 2xy - x &= 7 \\ x(2y - 1) &= 7 \\ \boxed{x} &= \boxed{\frac{7}{2y-1}} \end{aligned}$$

14. $3(2j - k) = 108$, for j

$$\begin{aligned} 6j - 3k &= 108 \\ 6j &= 108 + 3k \\ \boxed{j} &= \boxed{\frac{3k+108}{6}} \end{aligned}$$

11. $\frac{s}{2t} = 10$, for t

$$\begin{aligned} 2t \cdot \frac{s}{2t} &= 10 \cdot 2t \\ s &= 20t \\ \boxed{t} &= \boxed{\frac{s}{20}} \end{aligned}$$

13. $\left(\frac{d}{2} + \frac{f}{4}\right) = 6$, for f

$$\begin{aligned} 2d + f &= 24 \\ \boxed{f} &= \boxed{24 - 2d} \end{aligned}$$

15. $\frac{m}{n} + 5m = 20$, for m

$$\begin{aligned} m + 5mn &= 20n \\ m(1 + 5n) &= 20n \\ \boxed{m} &= \boxed{\frac{20n}{1+5n}} \end{aligned}$$

Key

Properties of Inequalities

Addition and Subtraction Properties for Inequalities	Multiplication and Division Properties for Inequalities
For any real numbers $a, b,$ and c : If $a < b,$ then $a + c < b + c$ and $a - c < b - c.$ If $a > b,$ then $a + c > b + c$ and $a - c > b - c.$	If you divide or multiple by a negative when solving an inequality remember to flip the inequality sign.

Example 1: Solve $2x + 4 > 36.$

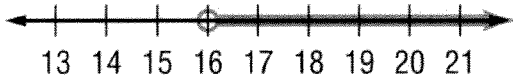
Graph the solution set on a number line.

$$2x + 4 - 4 > 36 - 4$$

$$2x > 32$$

$$x > 16$$

The solution set is $\{x \mid x > 16\}.$



Example 2: Solve $17 - 3w \geq 35.$

Graph the solution set on a number line.

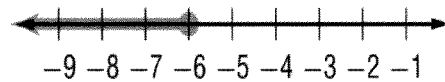
$$17 - 3w \geq 35$$

$$17 - 3w - 17 \geq 35 - 17$$

$$-3w \geq 18$$

$$w \leq -6$$

The solution set is $\{w \mid w \leq -6\}.$



Example 3 : GAMES After three quarters of the season has past, the Tigers have won 48 out of 72 games. How many of the remaining games must they win in order to win more than 70% of all their games this season?

Step 1: Understand/Define the Problem

Let x be the number of remaining games that the Tigers must win. The total number of games they will have won by the end of the season is $\frac{3}{4}(48 + x).$ They should win at least 70% of their games.

Step 2: Plan $\frac{3}{4}(48 + x) > 0.7(72)$

Step 3: Solve

$\frac{3}{4}(48 + x) > 0.7(72)$	Original Inequality
$48 + x > \frac{4}{3} \cdot 0.7(72)$	Multiply each side by $\frac{4}{3}.$
$48 + x > 67.2$	Simplify.
$x > 19.2$	Subtract 48 from each side.

Solve each inequality. Then graph the solution set on a number line.

16. $7(7a - 9) \leq 84$
 $7 \cdot 7a - 9 \leq 12$
 $7a \leq 21$
 $a \leq 3$

17. $3(9z + 4) > 35z - 4$
 $27z + 12 > 35z - 4$
 $16 > 8z$
 $2 > z$
 $z < 2$

18. $5(12 - 3n) < 165$
 $60 - 15n < 165$
 $-15n < 105$
 $n > -7$

19. $18 - 4k < 2(k + 21)$
 $18 - 4k < 2k + 42$
 $-24 < 6k$
 $-4 < k$
 $k > -4$

20. $4(b - 7) + 6 < 22$
 $4(b - 7) < 16$
 $b - 7 < 4$
 $b < 11$

21. $2 + 3(m + 5) \geq 4(m + 3)$
 $2 + 3m + 15 \geq 4m + 12$
 $3m + 17 \geq 4m + 12$
 $5 \geq m$
 $m \leq 5$

22. Jim makes \$5.75 an hour. Each week, 26% of his total pay is deducted for taxes. How many hours does Jim have to work if he wants his take-home pay to be at least \$110 per week? Write and solve an inequality for this situation.

Let $h = \#$ of hours

$$5.75h - (0.26)(5.75h) \geq 110$$

$$5.75h - 1.495h \geq 110$$

$$4.255h \geq 110$$

$h \geq 25.8519389...$

$h \geq 26$ hours