

Solving Non-Linear Systems

Name Key
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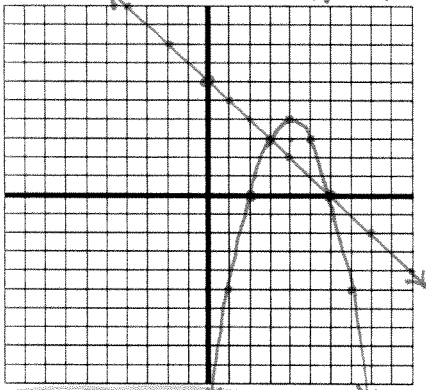
Solving Non-Linear Systems GRAPHICALLY

> To solve a NON-LINEAR SYSTEM of EQUATIONS by graphing, graph both equations on the same graph and find the points of intersection. The points of intersection are the points which will satisfy both equations and, therefore, are the solutions of the system! We can graph by hand and using the graphing calculator.

Solve each system graphically and state the points of intersection, as appropriate.

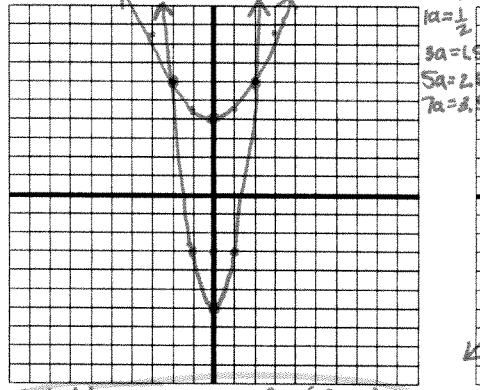
It may be necessary to use a graphing calculator to find an approximate solution.

1. $x + y = 6 \rightarrow y = -x + 6$
 $y = -(x-4)^2 + 4$ $V: (4, 4)$
 $a = -1$



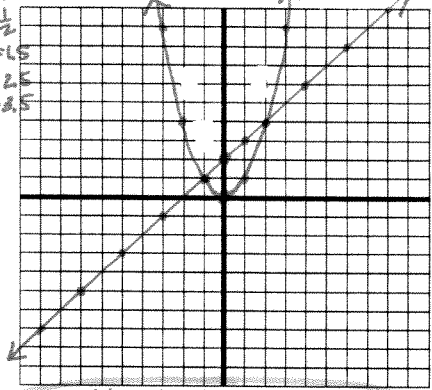
Solutions: (3, 3), (6, 0)

2. $2y = x^2 + 8$ $y = \frac{1}{2}x^2 + 4$ $V: (0, 4)$
 $y = 3x^2 - 6$ $V: (0, -6)$ $a = 3$



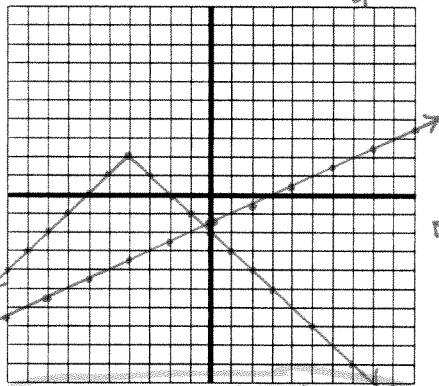
Solutions: (-2, 6), (2, 6)

3. $y = x^2$ $V: (0, 0)$ $a = 1$
 $y = x + 2$



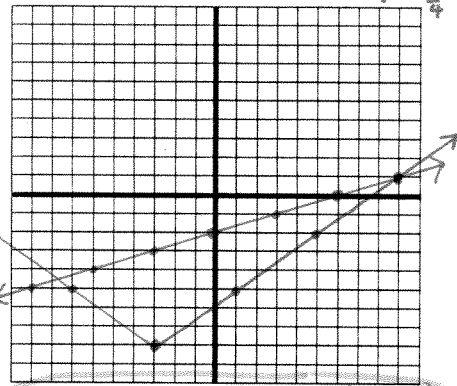
Solutions: (-1, 1), (2, 4)

4. line: $2y = x - 3$ $y = \frac{1}{2}x - \frac{3}{2}$
abs. value $y = -|x + 4| + 2$ $V: (-4, 2)$
 $a = -1$



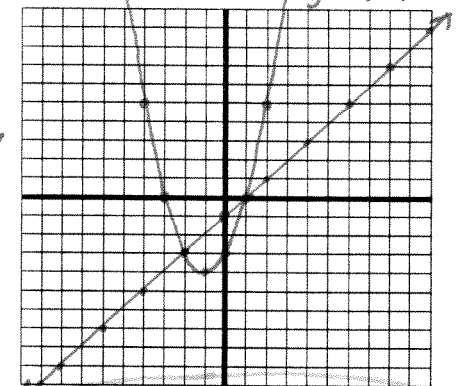
Solutions: (-15, -9), (-1/3, -1/3)

5. $-x + 3y = -6$ $3y = x - 6$ $y = \frac{1}{3}x - 2$
 $y = \frac{3}{4}|x + 3| - 8$ $V: (-3, -8)$
 $a = \frac{3}{4}$



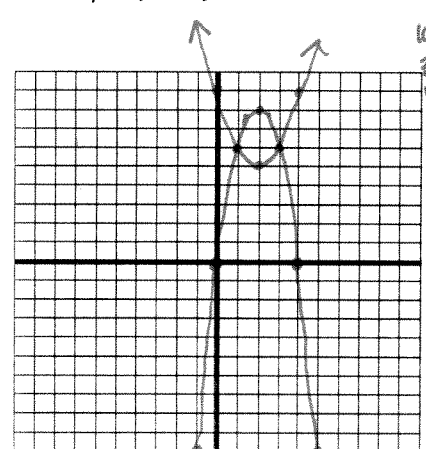
Solutions: (-7.62, -4.54), (9, 1)

6. $y + 4 = (x + 1)^2$ $y = (x + 1)^2 - 4$ $V: (-1, -4)$
 $x - y = 1$ $y = -x + 1$ $a = 1$
 $y = x - 1$



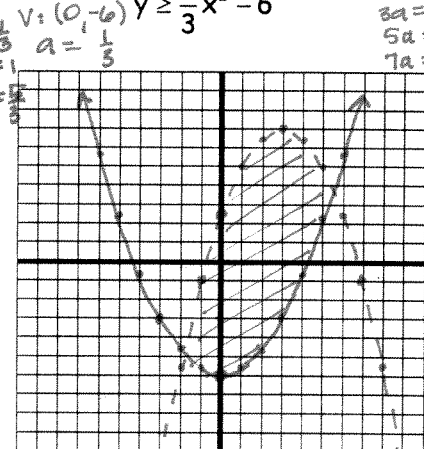
Solutions: (-2, -3), (1, 0)

7. $y = -2(x - 2)^2 + 8$ $V: (2, 8)$ $a = -2$
 $y = (x - 2)^2 + 5$

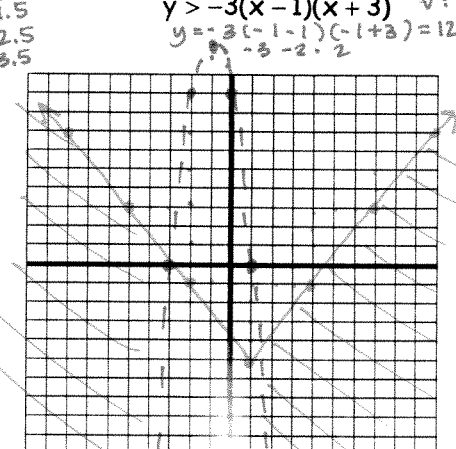


Solutions: (1, 6), (3, 6)

8. $y < \frac{1}{2}(x - 3)^2 + 7$ $V: (3, 7)$ $a = -\frac{1}{2}$
 $y \geq \frac{1}{3}x^2 - 6$ $V: (0, -6)$ $a = \frac{1}{3}$



9. $y \leq \frac{4}{3}|x - 1| - 5$
 $y > -3(x - 1)(x + 3)$ $V: (-1, 12)$
 $y = -3(-1-1)(-1+3) = 12$
 $-3 \cdot -2 \cdot 2 = 12$



$1a = -3$
 $3a = -9$
 $5a = -15$

Solving Non-Linear Systems ALGEBRAICALLY

> To solve a **NON-LINEAR SYSTEM** algebraically, use substitution or elimination.

Substitution: Solve one of the equations for one of the variables and substitute the result into the other equation.

Elimination: Multiply one or both equations by a constant that will cause either of the variables to cancel when added vertically.

Solve each system algebraically and state the points of intersection. You may verify your solutions using the graphing calculator.

$$\begin{aligned}
 1. \quad & y = x^2 - 4 \\
 & y = 3x \\
 & 3x = x^2 - 4 \\
 & x^2 - 3x - 4 = 0 \\
 & (x-4)(x+1) = 0 \\
 & x = 4 \quad x = -1 \\
 & y = 3x \quad y = 3x \\
 & y = 3(4) \quad y = 3(-1) \\
 & \boxed{(4, 12) \quad (-1, -3)}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & x + y = 6 \\
 & y = -(x-4)^2 + 4 \\
 & x - (x-4)^2 + 4 = 6 \\
 & x - (x^2 - 8x + 16) + 4 = 6 \\
 & x - x^2 + 8x - 16 + 4 = 6 \\
 & -x^2 + 9x - 12 = 6 \\
 & -x^2 + 9x - 18 = 0 \\
 & -(x^2 - 9x + 18) = 0 \\
 & -(x-3)(x-6) = 0 \\
 & x = 3 \quad x = 6 \\
 & x + y = 6 \quad x + y = 6 \\
 & 3 + y = 6 \quad 6 + y = 6 \\
 & y = 3 \quad y = 0 \\
 & \boxed{(3, 3) \quad (6, 0)}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & y = (x-1)^2 + 3 \\
 & 2x + y = 5 \\
 & 2x + (x-1)^2 + 3 = 5 \\
 & 2x + x^2 - 2x + 1 + 3 = 5 \\
 & x^2 + 4 = 5 \\
 & \sqrt{x^2} = \sqrt{1} \\
 & x = \pm 1 \\
 & x = 1 \quad x = -1 \\
 & 2x + y = 5 \quad 2x + y = 5 \\
 & 2(1) + y = 5 \quad 2(-1) + y = 5 \\
 & y = 3 \quad -2 + y = 5 \\
 & \quad \quad \quad y = 7 \\
 & \boxed{(1, 3) \quad (-1, 7)}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & x = 2(y+1)^2 - 6 \\
 & x + 3y = 5 \\
 & 2(y+1)^2 - 6 + 3y = 5 \\
 & 2(y^2 + 2y + 1) - 6 + 3y = 5 \\
 & 2y^2 + 4y + 2 - 6 + 3y = 5 \\
 & 2y^2 + 7y - 4 = 5 \\
 & 2y^2 + 7y - 9 = 0 \quad + \quad -18 \\
 & 2y^2 - 2y + 9y - 9 = 0 \quad + \quad -18 \\
 & 2y(y-1) + 9(y-1) = 0 \\
 & (y-1)(2y+9) = 0 \\
 & y = 1 \quad y = -\frac{9}{2} \\
 & x + 3y = 5 \quad x + 3y = 5 \\
 & x + 3(1) = 5 \quad x + 3(-\frac{9}{2}) = 5 \\
 & x = 2 \quad x - \frac{27}{2} = \frac{10}{2} \\
 & \quad \quad \quad x = \frac{37}{2} \\
 & \boxed{(2, 1) \quad (\frac{37}{2}, -\frac{9}{2})}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad & 3y = (x+3)^2 - 12 \\
 & x - y = -5 \quad x = y - 5 \\
 & 3y = (y-5+3)^2 - 12 \\
 & 3y = (y-2)^2 - 12 \\
 & 3y = y^2 - 4y + 4 - 12 \\
 & 3y = y^2 - 4y - 8 \\
 & 0 = y^2 - 7y - 8 \\
 & (y-8)(y+1) = 0 \\
 & y = 8 \quad y = -1 \\
 & x - y = -5 \quad x - y = -5 \\
 & x - 8 = -5 \quad x - (-1) = -5 \\
 & x = 3 \quad x + 1 = -5 \\
 & \quad \quad \quad x = -6 \\
 & \boxed{(3, 8) \quad (-6, -1)}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & x + 2y^2 = 4 \\
 & y = -x + 1 \\
 & x + 2(-x+1)^2 = 4 \\
 & x + 2(x^2 - 2x + 1) = 4 \\
 & x + 2x^2 - 4x + 2 = 4 \\
 & 2x^2 - 3x - 2 = 0 \quad + \quad -4 \\
 & 2x^2 - 4x + 1x - 2 = 0 \quad -4, 1 \\
 & 2x(x-2) + 1(x-2) = 0 \\
 & (x-2)(2x+1) = 0 \\
 & x = 2 \quad x = -\frac{1}{2} \\
 & y = -x + 1 \quad y = -x + 1 \\
 & y = -2 + 1 \quad y = -(-\frac{1}{2}) + 1 \\
 & y = -1 \quad y = \frac{1}{2} + 1 \\
 & \quad \quad \quad y = \frac{3}{2} \\
 & \boxed{(2, -1) \quad (-\frac{1}{2}, \frac{3}{2})}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad & x^2 = 8y \\
 & (x-2)^2 = 8y \\
 & x^2 = (x-2)^2 \\
 & x^2 = x^2 - 4x + 4 \\
 & 0 = -4x + 4 \\
 & 4x = 4 \\
 & x = 1 \\
 & x^2 = 8y \\
 & 1^2 = 8y \\
 & y = \frac{1}{8} \\
 & \boxed{(1, \frac{1}{8})}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & 2y = (x+5)^2 + 4 \\
 & y = x^2 + 10x + 25 \\
 & 2(x^2 + 10x + 25) = (x+5)^2 + 4 \\
 & 2x^2 + 20x + 50 = x^2 + 10x + 25 + 4 \\
 & 2x^2 + 20x + 50 = x^2 + 10x + 29 \\
 & x^2 + 10x + 21 = 0 \\
 & (x+7)(x+3) = 0 \\
 & x = -7 \quad x = -3 \\
 & y = x^2 + 10x + 25 \quad y = x^2 + 10x + 25 \\
 & y = (-7)^2 + 10(-7) + 25 \quad y = (-3)^2 + 10(-3) + 25 \\
 & y = 49 - 70 + 25 \quad y = 9 - 30 + 25 \\
 & y = 4 \quad y = 4 \\
 & \boxed{(-7, 4) \quad (-3, 4)}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & 6 - y = x^2 - 4x + 4 \\
 & y = -\frac{1}{2}(x-2)^2 + 4 \\
 & 6 - (-\frac{1}{2}(x-2)^2 + 4) = x^2 - 4x + 4 \\
 & 6 + \frac{1}{2}(x-2)^2 - 4 = x^2 - 4x + 4 \\
 & 6 + \frac{1}{2}(x^2 - 4x + 4) - 4 = x^2 - 4x + 4 \\
 & 6 + \frac{1}{2}x^2 - 2x + 2 - 4 = x^2 - 4x + 4 \\
 & \frac{1}{2}x^2 - 2x + 4 = x^2 - 4x + 4 \\
 & 0 = \frac{1}{2}x^2 - 2x \\
 & 0 = x(\frac{1}{2}x - 2) \\
 & x = 0 \quad \frac{1}{2}x - 2 = 0 \rightarrow x = 4 \\
 & y = -\frac{1}{2}(0-2)^2 + 4 \quad y = -\frac{1}{2}(4-2)^2 + 4 \\
 & y = -\frac{1}{2}(4) + 4 \quad y = -\frac{1}{2}(4) + 4 \\
 & y = -2 + 4 = 2 \quad y = -2 + 4 = 2 \\
 & \boxed{(0, 2) \quad (4, 2)}
 \end{aligned}$$