Quadratic Models

HOMEWORK Name Key
Date ________ Block ______

(1) ***Population Model*** The table shows the population of a town from 2000 through 2008. Find a quadratic model in standard form for the data. Assume that t is the number of years since 2000 and that P is measured in thousands of people.

on calc: Stat > Edit then L_1 and L_2

"x" L_1 Year, t	0	1	2	3	4	5	6	7	8
"y" L_2 Population, P	23.2	24	26.5	27.2	27.1	27.3	26.8	25.9	24

Find quadratic model using calc: Stat > Calc > QuadReg. $y = -0.242x^2 + 2.10x + 22.875$ → $P = -0.242t^2 + 2.10t + 22.875$

a. Estimate based on the data the town's population in 2010.

2010 → $t = 10$

$P = -0.242(10)^2 + 2.10(10) + 22.875$

$P = 19.675$ → $19.675 \cdot 1{,}000$ = **19675 people**

b. Estimate based on the data when the population was 18,500 and when it will be again.

18,500 → P = 18.5

$18.5 = -0.242t^2 + 2.10t + 22.875$

$0 = -0.242t^2 + 2.10t + 4.375$ ← Put this into Y= on the calc and find the positive x-int. (10.4)

x = −1.736004 ⇒ 1998

x 10.396004 ⇒

$t = 10.4$ → So the population will be about 18,500 in approximately 10.4 years from 2000 in the year 2010

c. How reliable is this method of estimation? Is it useful to make estimates in this way?

It is useful to make estimates this way, particularly if we don't extrapolate too far outside of the given data values. In other words, this method is fairly reliable if we make predictions within the years (0-8) closest to 2000 - 2008.

(2) A ball is dropped from a height of 10 feet. The height of the ball is modeled by $h(t) = -16t^2 + 10$, where h represents the height of the ball in feet after t seconds have elapsed.

a. How many seconds (to the nearest tenth) will it take the ball to reach the ground?

$h(t) = -16t^2 + 10$

$0 = -16t^2 + 10$ ← Put this into Y= on the calc and find the x-int / zero.

$t = 0.79$ ← The ball will reach the ground (height = 0) 0.79 seconds after it is dropped

b. At what time will the ball reach its highest point?

Find the maximum (vertex): From the calc. it looks like the y-int is the maximum. Algebraically $h = -16t^2 + 10$ → vertex "x" $= -\frac{b}{2a}$

"x" $= \frac{0}{2(-16)} = 0$

so the vertex IS at (0, 10)

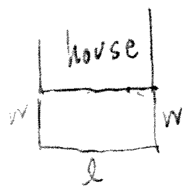
c. What is its highest point?

The highest point is when the ball is released at time zero.

vertex: (time, height)

(0, 10) ← The highest point is when the ball is released at 10 feet.

(3) Lorenzo has 48 feet of fencing to make a rectangular dog pen. If a house is used for one side of the pen, what would be the length and width for maximum area?



$A = lw$

$w + w + l = 48$

$2w + l = 48$

$A = lw$
$2w + l = 48$ } It's a system!

→ $l = 48 - 2w$

$A = lw$

$A = (48 - 2w)w$

$A = 48w - 2w^2$

$y = -2x^2 + 48x$ ← Put this in Y= on calc and find max/vertex. The vertex represents the length when the area is maximized (x) (y)

vertex/max: (12, 488) ← width at max area

let l = length and w = width

width at max area = 12 ft.

$w = 12$

$2w + l = 48$

$2(12) + l = 48$

$24 + l = 48$

$l = 24$

length = 24 ft
width = 12 ft

Key

4. Maura throws a baseball with an initial upward velocity of 60 feet per second. Given the model **$h(t)= v_ot - 16t^2$**, where h(t) is the height of an object in feet, v_o is the object's initial velocity in feet per second, and t is the time in seconds.

$y = 60t - 16t^2$

   a. Ignoring Maura's height, how long after she releases the ball will it hit the ground?

   x-intercept/zero: (3.75, 0)

   3.75 seconds

   b. How high was the ball after 2 seconds?

   when t/x = 2

   h(t)/y = 56

   (see the table on the calc.)

   56 feet

5. David threw a baseball into the air with an initial velocity of 80 feet per second. (use the model given in #4.)

$y = 80t - 16t^2$

   a. What is the maximum height the ball will travel?

   vertex/max: (2.5, 100) sec. feet

   100 feet

   b. When will the ball hit the ground?

   x-intercept/zero: (5, 0)

   5 seconds after it was thrown

6. An object is launched at 19.6 meters per second (m/s) from a 58.8-meter tall platform. The equation for the object's height *s* at time *t* seconds after launch is ***$s(t) = -4.9t^2 + 19.6t + 58.8$***, where *s* is in meters.

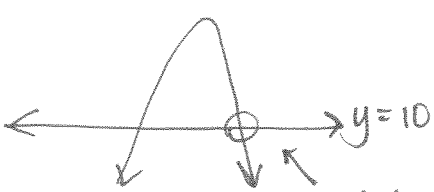
   a. When does the object strike the ground?

   x intercept/zero: (6, 0)

   6 seconds after it was launched

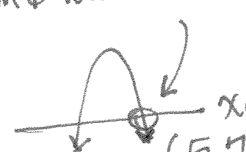
   b. How many seconds would it take for the object to be 10 meters above the ground?

   $10 = -4.9t^2 + 19.6t + 58.8$



   OR $s(t) = -4.9t^2 + 19.6t + 48.8$

   find the x-intercept/zero:



   approx. 5.74 seconds after it was launched